

Covariant holographic entanglement negativity

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Abstract

We conjecture a holographic prescription for the covariant entanglement negativity of d -dimensional conformal field theories dual to non static bulk AdS_{d+1} gravitational configurations in the framework of the AdS/CFT correspondence. Application of our conjecture to a AdS_3/CFT_2 scenario involving bulk rotating BTZ black holes exactly reproduces the entanglement negativity of the corresponding $(1+1)$ dimensional conformal field theories and precisely captures the distillable quantum entanglement. Interestingly our conjecture for the scenario involving dual bulk extremal rotating BTZ black holes also accurately leads to the entanglement negativity for the chiral half of the corresponding $(1+1)$ dimensional conformal field theory at zero temperature.

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1 Introduction

Quantum entanglement has emerged as one of the central issues in the subject of quantum information theory in recent times. This has inspired significant attention towards the characterization and the measurement of quantum entanglement in extended quantum many body systems. In this context the entanglement entropy has been established as one of the crucial measures for quantum entanglement in bipartite quantum systems. For this it is required to partition a quantum system into the subsystem- A and the rest of the system denoted as A^c , which is described by a pure quantum state $|\psi\rangle$. In such a scenario, the density matrix of the full system is given by $\rho = |\psi\rangle\langle\psi|$ and the entanglement entropy which measures the quantum entanglement between the subsystem- A and it's complement A^c is defined as

$$S_A = -Tr(\rho_A \log \rho_A), \quad (1)$$

where ρ_A is the reduced density matrix obtained by tracing out the degrees of freedom of A^c i.e $\rho_A = Tr_{A^c}(\rho)$. The issue of computing the entanglement entropy for a bipartite quantum many body system is extremely non trivial as it involves the determination of the eigenvalues of the infinite dimensional density matrix- ρ_A . However, in a $(1+1)$ -dimensional quantum field theory with conformal symmetry this problem may be resolved using the *replica technique* as demonstrated by Calabrese and Cardy in [1, 2]. In this technique, the quantity $Tr(\rho_A^n)$ for any non-negative integer n corresponds to the partition function on a n -sheeted Riemann surface ($\mathcal{Z}_n(A)$) with branch points at the boundaries between the subsystems A and A^c [1]. The moments of the reduced density matrix $Tr(\rho_A^n)$ are then related to the entanglement entropy of a $(1+1)$ - dimensional conformal field theory (CFT) as follows

$$S_A = -\frac{\partial}{\partial n} \log [Tr(\rho_A^n)] \Big|_{n=1} = \log \mathcal{Z} - \frac{\partial}{\partial n} \log \mathcal{Z}_n(A) \Big|_{n=1}. \quad (2)$$

Here, \mathcal{Z} corresponds to the partition function of CFT_{1+1} on a single sheet of a n -sheeted Riemann surface. The quantity $\mathcal{Z}_n(A)$ in the above equation is related to the two point function of certain branch-point twist and anti-twist fields in the corresponding CFT_{1+1} which may then be computed in a straight forward way to obtain the entanglement entropy. Note that this replica procedure is very general and applicable to both time independent and time dependent scenarios. For time dependent states in CFT_{1+1} the reduced density matrix ρ_A in eq.(2) has to be replaced by the time dependent $\rho_A(t) = Tr_{A^c}(\rho(t))$ where the full density matrix $\rho(t)$ evolves according to the well known von-Neumann equation. Naturally this leads to a time dependent two point function of the twist and the anti/twist fields which describes the evolution of the entanglement entropy in CFT_{1+1} [3]. This remarkable progress in studying the time evolution of entanglement entropy in $(1+1)$ -dimensional quantum field theories has inspired focused attention in diverse areas such as quantum quenches, thermalization and quantum phase transitions [4–6].

It is well known in quantum information theory that the entanglement entropy is not a suitable measure to obtain the distillable quantum entanglement for a bipartite quantum system in a mixed state¹. For instance, when the bipartite quantum system is at a finite temperature, the entanglement entropy receives contribution from both the classical and the quantum correlations as discussed in [2]. Moreover, the entanglement entropy for such bipartite quantum systems is dominated by the thermal entropy of the subsystem at large temperatures. This relates to an important issue in quantum information theory termed as *purification* which is a technique of defining justifiable measures that capture the distillable quantum entanglement for a bipartite

¹Note that the system may be in a mixed state in a variety of physical situations. A state may be mixed as a result of the interaction between the system and its environment or in the finite temperature case due to the interaction between the system and the heat reservoir. It may also occur if the system is in a degenerate ground state as in the case of a CFT which is dual to an extremal *AdS* black hole. We will elaborate more about this case in one of the forthcoming sections.

quantum system in a mixed state. In this regard, Vidal and Werner in [7] proposed a new measure termed as the *entanglement negativity* which captures the distillable quantum entanglement when a bipartite quantum system ($A_1 \cup A_2$) is in a mixed state. More precisely, the definition of the entanglement negativity involves the above mentioned *purification* procedure which requires one to embed the system under consideration in a larger system such that the full system is described by a pure state. Following this, the degrees of freedom of the larger system are traced out to obtain the mixed state density matrix of the required system. Consider a tripartite system divided into A_1, A_2 such that $A = A_1 \cup A_2$ and the rest of the system denoted by A^c , then the entanglement negativity of the subsystem A_1 may be defined as follows

$$\mathcal{E} \equiv \ln [Tr | \rho_A^{T_2} |]. \quad (3)$$

Where $\rho_{A_1 \cup A_2} = Tr_{A^c}(\rho)$ is the reduced density matrix of the subsystem $A = A_1 \cup A_2$ and the super-script T_2 corresponds to the operation of the partial transpose over the subsystem A_2 . In this regard, if $|q_i^1\rangle$ and $|q_i^2\rangle$ represent the bases of Hilbert space corresponding to the subsystems A_1 and A_2 respectively, then the partial transpose with respect to A_2 degrees of freedom is expressed as

$$\langle q_i^1 q_j^2 | \rho_{A_1 \cup A_2}^{T_2} | q_k^1 q_l^2 \rangle = \langle q_i^1 q_l^2 | \rho_{A_1 \cup A_2} | q_k^1 q_j^2 \rangle, \quad (4)$$

Furthermore, the entanglement negativity was shown to obey certain important properties such as monotonicity and non convexity in [8]. Note that similar to the case of entanglement entropy, obtaining the entanglement negativity for extended quantum systems is extremely complex as this also involves the evaluation of the eigen values of an infinite dimensional density matrix. However, for the special case of $(1+1)$ - dimensional CFT at a finite temperature this becomes possible through a variant of the replica technique which was proposed in [9]. There the authors Calabrese et al. showed that the entanglement negativity for a CFT_{1+1} at a finite temperature may be defined through the logarithm of a certain four point function of the twist and anti-twist fields. They also showed that this quantity measures the distillable quantum entanglement through the removal of the thermal contribution.

Further advances in understanding different aspects of the quantum entanglement in the context of higher dimensional strongly coupled conformal field theories may be attributed to the *AdS/CFT* correspondence or the *gauge/gravity duality* [10–13]. This holographic correspondence relates a weakly coupled theory of gravity in a $(d+1)$ -dimensional bulk *AdS* space time and a d dimensional strongly coupled conformal field theory on the boundary of the space time. Note that this correspondence works best in the large- N limit which for a generic CFT translates to the large central charge limit [14]. In this context, Ryu and Takayanagi proposed a holographic prescription for obtaining the entanglement entropy for a strongly coupled boundary conformal field theories at finite temperatures in arbitrary dimensions [15, 16]. Their prescription directly relates the entanglement entropy S_A for a region A (enclosed by the boundary ∂A) in the (d) -dimensional boundary quantum field theory to the area of the co-dimension two static minimal surface (denoted by γ_A) extending from the boundary ∂A of the region A into the $(d+1)$ -dimensional bulk which may be expressed as

$$S_A = \frac{Area(\gamma_A)}{(4G_N^{(d+1)})}. \quad (5)$$

Here, $G_N^{(d+1)}$ is the gravitational constant of the bulk space time. This conjecture has seen tremendous success in exploring various phenomenon involving the quantum entanglement in higher dimensional CFTs in the context of the *AdS/CFT* correspondence [17, 18]. However, note that the Ryu-Takayanagi conjecture elucidated above is applicable only for the conformal field theories whose holographic duals are described by the static solutions in the bulk *AdS* space time. The reason for this is intricately related to the ambiguity in defining minimal surfaces in a gravitational configuration with a Lorentzian signature. We will discuss more about this in

the next section. This raises the critical issue of determining the time evolution of entanglement entropy in higher dimensional quantum field theories with conformal symmetry. Recently, the authors Hubeny et. al in [19] have advanced a covariant holographic prescription which may be employed to obtain the entanglement entropy of conformal field theories which are dual to non-static solutions of the weakly coupled theory of gravity in the bulk AdS space time. Their covariant holographic prescription involves the basic idea of exploiting the light-sheet construction of the covariant entropy bounds of Bousso [20–22]. These light-sheets provide a way to single out a co-dimension two spacelike surface of the bulk AdS space time whose area gives a natural bound to the entropy passing through the light-sheet of the surface. The explicit realization of a covariant holographic proposal for the entanglement entropy has led to an intense investigation involving the study of the time evolution of the entanglement entropy in processes such as quantum quenches and thermalization of CFTs dual to various gravitational configurations in the context of holography [23–25].

The above discussion naturally led to the crucial issue of establishing a precise and elegant holographic prescription for the entanglement negativity of conformal field theories at finite temperatures in the AdS/CFT framework. A recent conjecture for such a holographic prescription for the entanglement negativity was proposed by us Chaturvedi, Malvimat and Sengupta (CMS) in [26, 27]. It could be demonstrated that the holographic entanglement negativity (\mathcal{E}) of a subsystem- A is proportional to a particular algebraic sum of the co-dimension two static minimal surfaces in the dual bulk $(d+1)$ -dimensional AdS space time as follows

$$\mathcal{E} = \lim_{B \rightarrow A^c} \frac{3}{16G_N^{d+1}} [2\mathcal{A}_A + \mathcal{A}_{B_1} + \mathcal{A}_{B_2} - \mathcal{A}_{A \cup B_1} - \mathcal{A}_{A \cup B_2}]. \quad (6)$$

Where \mathcal{A}_γ is the area of the minimal surface anchored on the corresponding subsystem $\gamma = \{A, B_1, B_2, A \cup B_1, A \cup B_2\}$ and the limit $B \rightarrow A^c$ in the above expression corresponds to extending the two large finite subsystems B_1 and B_2 on either side of (A) to infinity such that in this limit $B_1 \cup B_2$ denoted as B is the rest of the system A^c . Notice that the algebraic sum of the areas in the above equation may be re-expressed as the sum of the holographic mutual information between the subsystem- (A, B_1) and (A, B_2) as follows

$$\begin{aligned} \mathcal{E} &= \lim_{B \rightarrow A^c} \frac{3}{4} [\mathcal{I}(A, B_1) + \mathcal{I}(A, B_2)] \\ \mathcal{I}(A, B_i) &= S_A + S_{B_i} - S_{A \cup B_i} = \frac{1}{4G_N^{(d+1)}} (\mathcal{A}_A + \mathcal{A}_{B_i} - \mathcal{A}_{A \cup B_i}), \end{aligned} \quad (7)$$

where, $i = \{1, 2\}$ for the corresponding subsystems B_1 and B_2 . In the AdS_3/CFT_2 scenario, we employed our conjecture to compute the finite temperature entanglement negativity of a CFT_{1+1} from the algebraic sum of geodesics in the bulk which is an Euclidean BTZ black hole. We demonstrated that the holographic entanglement negativity obtained from our conjecture matches exactly with the CFT_{1+1} result in the large central charge limit. Furthermore, in an another communication [27] we applied our conjecture to obtain holographic entanglement negativity of d -dimensional CFTs at finite temperatures which are dual to bulk AdS_{d+1} -Schwarzschild black holes. Remarkably it could be demonstrated that in both the cases the holographic entanglement negativity precisely captured the distillable quantum entanglement at all temperatures. This naturally constituted a strong evidence for the universality of our conjecture.

Notice however that our holographic conjecture proposed and studied in [26, 27] is only applicable to conformal field theories which are dual to static bulk AdS black holes due to the ambiguity in the definition of minimal surfaces in non static space time geometries. In this article we propose such a holographic conjecture for the covariant entanglement negativity of d dimensional conformal field theories dual to bulk non static AdS_{d+1} gravitational configurations. Subsequently using our conjecture, we obtain the covariant entanglement negativity of a CFT_{1+1} at a finite temperature dual to a bulk rotating BTZ black hole in a AdS_3/CFT_2

scenario. Furthermore we also apply our conjecture to obtain the same for a CFT_{1+1} dual to an extremal rotating bulk BTZ black hole. Interestingly in both the cases described above the covariant holographic entanglement negativity obtained from the bulk calculations could be exactly reproduced for that computed from the corresponding CFT_{1+1} in the large central charge c limit. Furthermore as earlier for static bulk configurations, it is observed that the holographic entanglement negativity precisely leads to the distillable quantum entanglement.

This article is organized as follows. In section-2 we review the HRT prescription for the covariant holographic entanglement entropy in the AdS_{d+1}/CFT_d scenario. In section-3 we propose our covariant holographic conjecture for the entanglement negativity of d dimensional CFTs which are dual to non static AdS_{d+1} space times. In section-4 we use our prescription to compute the entanglement negativity of a CFT_{1+1} dual to a rotating non-extremal BTZ black hole background. In section-5 we compute the entanglement negativity of a finite temperature CFT_{1+1} with angular momentum and demonstrate that the large- c limit of this result matches exactly with that obtained from bulk using our holographic prescription. In section-6 we compute the covariant holographic entanglement negativity of the CFT_{1+1} dual to the extremal rotating BTZ black hole and show that entanglement negativity captures the distillable quantum entanglement at all temperatures. Subsequently, in section-7 we compute the entanglement negativity in the dual CFT and show that the result once again matches exactly with the bulk computation in the large central charge (c) limit.

2 Review of the covariant holographic entanglement entropy

As mentioned earlier the replica procedure provided by Calabrese et al. is a systematic method to obtain the entanglement entropy of $(1 + 1)$ -dimensional conformal field theories. As there is no such direct procedure to evaluate the entanglement entropy in the higher dimensional CFTs one must make use of the holographic conjecture by Ryu and Takayanagi which we briefly described in the introduction. Using their holographic prescription, it was possible to obtain the entanglement entropy of various higher dimensional CFTs in the large central charge limit [28–34]. However unlike the replica procedure which is applicable to both time dependent and time independent scenarios in CFT_{1+1} , the Ryu and Takayanagi conjecture is valid only for the CFTs which are dual to static AdS space time backgrounds. The reason for this is as follows. The Ryu and Takayanagi conjecture relates the entanglement entropy of a subsystem in CFT_d to a minimal surface in the corresponding bulk AdS_{d+1} space time. These minimal surfaces are well defined in an Euclidean geometry whereas in a Lorentzian geometry one may always contract the spacelike surface along the time direction to reduce the area of the minimal surface to arbitrarily small values. In a time independent scenario it is always possible to perform a Wick rotation leading to an Euclidean AdS geometry in the bulk, but this is not possible anymore while describing time dependent phenomena in the boundary conformal field theory. Therefore, the Ryu-Takayanagi conjecture is unsuitable for describing the time evolution of the entanglement entropy in CFTs. This issue was resolved by Hubeny, Rangamani and Takayanagi(HRT) in [19] where the authors provided a covariant prescription for the holographic entanglement entropy of a d -dimensional CFT. We briefly review their proposal here.

The HRT conjecture for the covariant holographic entanglement entropy of a d dimensional CFT dual to a non-static asymptotically AdS_{d+1} space time [19], was inspired by the light sheet construction for the covariant Bousso bound [20, 22, 35]. Consider a co-dimension two spacelike hypersurface \mathcal{S} , a light sheet $L_{\mathcal{S}}$ for this \mathcal{S} is defined as a null hypersurface that is bounded by \mathcal{S} and is generated by null geodesic congruences whose expansion is non-positive definite. According to the Bousso bound, the thermodynamic entropy $S_{L_{\mathcal{S}}}$ through a light sheet $L_{\mathcal{S}}$ is bounded by the area of \mathcal{S} in Planck units

$$S_{L_{\mathcal{S}}} \leq \frac{Area(\mathcal{S})}{4G_N}. \quad (8)$$

If L_- and L_+ represent the past and the future light sheets respectively, then the corresponding null expansions denoted by θ_{\pm} obey the inequality $\theta_{\pm} \leq 0$. The authors in [19] argue that the holographic entanglement entropy saturates the above mentioned Bousso bound. They consider a d -dimensional strongly coupled conformal field theory living on the boundary of $(d+1)$ -dimensional AdS space time. The asymptotic boundary of the AdS_{d+1} space time where the conformal field theory is situated is divided into two time dependent regions A_t and A_t^c . The boundary of the region A_t is denoted as ∂A_t . Following this, the past and future light sheets of a for spacelike surface ∂A_t , denoted as ∂L_+ and ∂L_- are constructed. One is then required to extend ∂L_+ and ∂L_- into the bulk in such a way that their extensions denoted by L_+ and L_- respectively, are also light sheets corresponding to a $(d-1)$ -dimensional spacelike surface $\mathcal{Y}_t = L_+ \cap L_-$ which is anchored to ∂A_t . According to the HRT proposal, out of all such surfaces \mathcal{Y}_t the holographic entanglement entropy of the region A_t is given by the surface which has the minimum area (\mathcal{Y}_t^{min}). The authors also showed that this surface (\mathcal{Y}_t^{min}) is also the extremal surface (\mathcal{Y}_t^{ext}) anchored to ∂A_t and the null expansions for this spacelike hypersurface vanish (i.e $\theta_{\pm} = 0$).

$$S_{A_t} = \frac{Area(\mathcal{Y}_t^{min})}{4G_N^{(d+1)}} = \frac{Area(\mathcal{Y}_t^{ext})}{4G_N^{(d+1)}}. \quad (9)$$

Notice that in the AdS_3/CFT_2 scenario the extremal area in the above equation is replaced by the extremal geodesic length. The authors verified their prescription in the AdS_3/CFT_2 context as follows. They first computed the entanglement entropy of a $(1+1)$ -dimensional finite temperature CFT with a conserved angular momentum using the standard replica technique and demonstrated that it is given by

$$S_A = \frac{c}{6} \log \left[\frac{\beta_+ \beta_-}{\pi^2 a^2} \sinh \left(\frac{\pi \ell}{\beta_+} \right) \sinh \left(\frac{\pi \ell}{\beta_-} \right) \right]. \quad (10)$$

Where, $\beta_+ = \beta(1 + \Omega)$ and $\beta_- = \beta(1 - \Omega)$ are the left and the right moving temperatures the CFT_{1+1} , Ω is the angular velocity, ℓ is the length of the subsystem- A and a is the UV cut-off for the field theory. They then used the above mentioned holographic prescription to obtain the entanglement entropy of the CFT dual to the rotating BTZ black hole background and demonstrated that the result matches exactly with the CFT result given by eq.(10).

3 Covariant holographic entanglement negativity conjecture

In this section we propose a covariant prescription for the holographic entanglement negativity in the AdS_{d+1}/CFT_d scenario. In order to do this we first briefly review our recently conjectured prescription for the holographic entanglement negativity of CFTs dual to static AdS space time configurations [27]. The d -dimensional conformal field theory on the asymptotic boundary a static AdS_{d+1} space time is partitioned into two regions A and its complement A^c . The region B within the complement A^c consists of two large but finite disjoint subsystems B_1 and B_2 on either side of A , such that $B = B_1 \cup B_2$. The holographic entanglement negativity which quantifies the entanglement between a subsystem- A and the rest of the system A^c , is given by an algebraic sum of the areas of the static minimal surfaces in the bulk as follows

$$\mathcal{E} = \lim_{B \rightarrow A^c} \frac{3}{16G_N^{d+1}} [2\mathcal{A}_A + \mathcal{A}_{B_1} + \mathcal{A}_{B_2} - \mathcal{A}_{A \cup B_1} - \mathcal{A}_{A \cup B_2}]. \quad (11)$$

Where \mathcal{A}_{γ} represents the co dimension two static minimal surface anchored on the corresponding subsystem γ and the limit $B \rightarrow A^c$ has to be interpreted as extending the subsystems B_1 and B_2 to infinity such that in this limit B is A^c . Note that the above equation may be re-expressed in terms of the holographic mutual information denoted by $\mathcal{I}(A, B_1)$ and $\mathcal{I}(A, B_2)$ as given by eq.(7).

However the above expression for the holographic entanglement negativity is valid only for CFT_d dual to static AdS_{d+1} gravitational configurations such as the AdS_{d+1} - Schwarzschild black holes. For a CFT_d dual to a non static bulk AdS_{d+1} gravitational configuration it is possible to reduce the area of the minimal surface to zero which leads to a degenerate situation. Notice from [19] that a similar issue also occurs for the case of the entanglement entropy and is resolved through the replacement of the area of the minimal surface with that of the extremal surface. It is now clear that to obtain the covariant holographic entanglement negativity for a CFT_d dual to a non static AdS_{d+1} gravitational configuration, the areas of the static minimal surfaces (\mathcal{A}) in eq.(11) is required to be replaced by that of the corresponding extremal surfaces (\mathcal{Y}^{ext}). The above discussion leads to the precise holographic conjecture for the covariant entanglement negativity

$$\mathcal{E} = \lim_{B \rightarrow A^c} \frac{3}{16G_N^{d+1}} [2\mathcal{Y}_A^{ext} + \mathcal{Y}_{B_1}^{ext} + \mathcal{Y}_{B_2}^{ext} - \mathcal{Y}_{A \cup B_1}^{ext} - \mathcal{Y}_{A \cup B_2}^{ext}]. \quad (12)$$

where \mathcal{Y}_γ^{ext} is the area of the extremal surface anchored on the corresponding subsystem γ . The above equation may once again be re-expressed as the holographic mutual information between pairs of subsystems (A, B_1) and (A, B_2) as

$$\mathcal{E} = \lim_{B \rightarrow A^c} \left[\frac{3}{4} (\mathcal{I}(A, B_1) + \mathcal{I}(A, B_2)) \right], \quad (13)$$

$$\mathcal{I}(A, B_1) = S_A + S_{B_1} - S_{A \cup B_1} = \frac{1}{4G_N^3} (\mathcal{Y}_A^{ext} + \mathcal{Y}_{B_1}^{ext} - \mathcal{Y}_{A \cup B_1}^{ext}), \quad (14)$$

$$\mathcal{I}(A, B_2) = S_A + S_{B_2} - S_{A \cup B_2} = \frac{1}{4G_N^3} (\mathcal{Y}_A^{ext} + \mathcal{Y}_{B_2}^{ext} - \mathcal{Y}_{A \cup B_2}^{ext}), \quad (15)$$

We put our proposal to test in the forthcoming sections in the AdS_3/CFT_2 context. In the next section, we compute the entanglement negativity of a subsystem $(1+1)$ -dimensional CFT with rotation and discuss its large- c limit and in the subsequent section we use the above described proposal of ours to evaluate the same from the bulk BTZ black hole background which is a stationary space time.

4 Covariant holographic entanglement negativity of a CFT_{1+1} dual to a rotating non-extremal BTZ

In this section we compute the covariant holographic entanglement negativity of a CFT_{1+1} dual to a $(2+1)$ -dimensional rotating non-extremal BTZ black hole in the AdS_3/CFT_2 scenario (we will consider the extremal black hole case in a latter section as it is special). Note that this corresponds to a CFT_{1+1} with a conserved angular momentum and at a finite temperature. The metric for a rotating BTZ black hole is given by

$$ds^2 = -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2 dr^2}{(r^2 - r_+^2)(r^2 - r_-^2)} + r^2 (d\phi - \frac{r_+ r_-}{r^2} dt)^2. \quad (16)$$

Here, we have set the the AdS length scale to unity ($R = 1$) and r_- and r_+ in the above equation correspond to the inner and outer horizon radius of the black hole. The mass M , the angular momentum J , the Hawking temperature T_H and the angular velocity Ω of the black hole may be expressed in terms of r_- and r_+ as follows

$$M = r_+^2 + r_-^2, \quad J = 2r_+ r_-, \quad \beta = \frac{1}{T_H} = \frac{2\pi r_+}{r_+^2 - r_-^2}, \quad \Omega = \frac{r_-}{r_+}, \quad \beta_{\pm} = \beta(1 - \Omega). \quad (17)$$

Notice that for a BTZ black hole ϕ is periodic i.e $\phi \sim \phi + 2\pi$ where as for a BTZ black string $\phi \in \mathbb{R}$. As the computation of the holographic entanglement entropy involves dual bulk

black holes with planar horizons in the AdS/CFT scenario it is required to consider a bulk BTZ black string which is a stationary gravitational configuration given by the metric in eq.(16). Hence as discussed in the section-3 the covariant proposal of HRT must be used to compute the holographic entanglement entropy. We first briefly review their covariant proposal [19] which involves the following co-ordinate transformation as a first step,

$$w_{\pm} = \sqrt{\frac{r^2 - r_+^2}{r^2 - r_-^2}} e^{\frac{2\pi}{\beta_{\pm}}(\phi \pm t)} \equiv X \pm T, \quad (18)$$

$$Z = \sqrt{\frac{r_+^2 - r_-^2}{r^2 - r_-^2}} e^{(r_+ \phi_i - t r_-)}. \quad (19)$$

Where $w_{\pm} = X \pm T$ are the light cone coordinates and (X, T, Z) are the the Poincaré coordinates. The above transformation maps the metric of the BTZ black hole given in eq.(16) to the Poincaré metric of the pure AdS_3 given by

$$ds^2 = \frac{dw_+ dw_- + dZ^2}{Z^2} \equiv \frac{-dT^2 + dX^2 + dZ^2}{Z^2}. \quad (20)$$

The length of the required spacelike geodesics is well known for the pure AdS_3 space time and leads to the following expression for the entanglement entropy

$$S_{\gamma} = \frac{\mathcal{L}_{\gamma}}{4G_N^{(3)}} = \frac{c}{6} \log \left[\frac{(\Delta X)^2}{\varepsilon_i \varepsilon_j} \right], \quad (21)$$

$$\varepsilon_i = \sqrt{\frac{r_+^2 - r_-^2}{r_{\infty}^2}} e^{(r_+ \phi_i - t_0 r_-)}. \quad (22)$$

Where S_{γ} represents the entanglement entropy of a subsystem γ which is a spacelike interval $[\phi_i, \phi_j]$ on the boundary conformal field theory, \mathcal{L}_{γ} is the length of spacelike geodesic anchored to the boundary of the subsystem γ and r_{∞} is the infrared cut-off for the bulk in the BTZ coordinates where as ε_i is the same in Poincaré coordinates. Notice that the constant time slice is taken along $t = t_0$. If we re-express the eq.(21) in terms of BTZ coordinates then we get the required entanglement entropy [19] as

$$S_{\gamma} = \frac{c}{6} \log \left[\frac{\beta_+ \beta_-}{\pi^2 a^2} \sinh \left(\frac{\pi |\phi_i - \phi_j|}{\beta_+} \right) \sinh \left(\frac{\pi |\phi_i - \phi_j|}{\beta_-} \right) \right], \quad (23)$$

where a is the UV cut-off for the boundary CFT related to the bulk infrared cut-off ($a \sim \frac{1}{r_{\infty}}$).

Having briefly reviewed the method to obtain the expression for the holographic entanglement entropy, we now use our holographic conjecture to compute the covariant holographic entanglement negativity for a CFT_{1+1} dual to a rotating non-extremal BTZ black hole. As discussed in the previous section the covariant holographic entanglement negativity is in terms of mutual information between different pairs of intervals and is given by

$$\mathcal{E} = \lim_{B \rightarrow A^c} \left[\frac{3}{4} (\mathcal{I}(A, B_1) + \mathcal{I}(A, B_2)) \right], \quad (24)$$

$$\mathcal{I}(A, B_1) = S_A + S_{B_1} - S_{A \cup B_1} = \frac{1}{4G_N^3} (\mathcal{L}_A + \mathcal{L}_{B_1} - \mathcal{L}_{A \cup B_1}), \quad (25)$$

$$\mathcal{I}(A, B_2) = S_A + S_{B_2} - S_{A \cup B_2} = \frac{1}{4G_N^3} (\mathcal{L}_A + \mathcal{L}_{B_2} - \mathcal{L}_{A \cup B_2}). \quad (26)$$

In order to do the entanglement negativity of the subsystem A we make the following identification for the points $(\phi_1, \phi_2, \phi_3, \phi_4) \equiv (-L, u, v, L)$ which implies that the required subsystems

A , B_1 , B_2 correspond to the intervals given by $[u, v]$, $[-L, u]$ and $[v, L]$ respectively. We use the expression given by eq.(23) to obtain the entanglement entropy of each of these intervals i.e S_A , S_{B_i} and $S_{A \cup B_i}$ ($i = 1, 2$). These are then substituted in eq.(24) and the limit $B \rightarrow A^c$ (which corresponds to the limit $L \rightarrow \infty$) is taken to obtain the entanglement negativity as

$$\mathcal{E} = \frac{c}{4} \log \left[\frac{\beta_+ \beta_-}{\pi^2 a^2} \sinh \left(\frac{\pi \ell}{\beta_+} \right) \sinh \left(\frac{\pi \ell}{\beta_-} \right) \right] - \frac{\pi c \ell}{2\beta(1 - \Omega^2)}. \quad (27)$$

Re-expressing the above equation in terms of the entanglement entropy and the thermodynamic entropy of subsystem-A, we get

$$\mathcal{E} = \frac{3}{2} \left[S_A - S_A^{th} \right]. \quad (28)$$

The above equation clearly establishes that the covariant holographic entanglement negativity captures the distillable quantum entanglement by subtracting away the contribution from the thermal correlations.

5 Entanglement negativity in a CFT_{1+1} with angular momentum and at a finite temperature

In this section, we compute the entanglement negativity of a bipartite quantum system in a $(1+1)$ -dimensional CFT with a conserved angular momentum and at a finite non-zero temperature. We will demonstrate that the entanglement negativity precisely captures the distillable entanglement and also show that in the large- c limit it matches exactly with the result obtained in the previous section using our covariant holographic conjecture involving a bulk rotating non-extremal BTZ black hole. As described in the introduction a modification of the replica technique for the entanglement entropy was developed by Calabrese, Cardy and Tonni in [9] to obtain the entanglement negativity of a bipartite quantum system described by a $(1+1)$ -dimensional conformal field theory, which is expressed as

$$\mathcal{E} = \lim_{n_e \rightarrow 1} \log(\text{Tr}[(\rho_A^T)^{n_e}]). \quad (29)$$

The full system is partitioned into subsystem-A which is an interval $[u, v]$ and the rest of the system denoted as A^c . The quantity ρ_A^T is the reduced density matrix of the subsystem-A partial transposed over A^c (For the details of how to obtain the above expression for the entanglement negativity of a bipartite quantum system from its definition for the tripartite configuration given by eq.(3), see [9] or the appendix of [27]). Notice that this definition is valid only when the parity of n is even ($n = n_e$) i.e the above definition has to be interpreted as an analytic continuation of an even sequence in n_e to $n_e \rightarrow 1$. The authors in [9] demonstrated that for a CFT_{1+1} at a finite temperature the quantity $(\text{Tr}[(\rho_A^T)^{n_e}])$ is related to a particular four point function of the twist and the anti-twist fields, resulting in the following expression for the entanglement negativity

$$\mathcal{E} = \lim_{L \rightarrow \infty} \lim_{n_e \rightarrow 1} \log \left[\langle \mathcal{T}_{n_e}(-L) \overline{\mathcal{T}}_{n_e}^2(u) \mathcal{T}_{n_e}^2(v) \overline{\mathcal{T}}_{n_e}(L) \rangle_\beta \right], \quad (30)$$

where $\langle \dots \rangle_\beta$ in the above equation indicates that this four point function has to be evaluated in the finite temperature CFT_{1+1} on an infinite cylinder of circumference β ($\beta = 1/T$ where T is the temperature). In the above equation \mathcal{T}_{n_e} and $\overline{\mathcal{T}}_{n_e}$ are the twist and the anti-twist fields which are primary operators with the scaling dimension Δ_{n_e} , whereas $\mathcal{T}_{n_e}^2$ and $\overline{\mathcal{T}}_{n_e}^2$ are the twist and the anti-twist fields are also primary operators but with the scaling dimension $\Delta_{n_e}^{(2)}$. These scaling dimensions are given by

$$\Delta_{n_e} = \frac{c}{12} \left(n_e - \frac{1}{n_e} \right), \quad (31)$$

$$\Delta_{n_e}^{(2)} = 2\Delta_{\frac{n_e}{2}} = \frac{c}{6} \left(\frac{n_e}{2} - \frac{2}{n_e} \right), \quad (32)$$

The authors in [9] computed the entanglement negativity using the above eq.(30) to illustrate that this quantity is a precise measures of the distillable quantum entanglement through the removal of the thermal contributions.

In present case we consider a CFT_{1+1} at a finite temperature, having a conserved angular momentum. The partition function of this CFT is given by

$$Z(\beta) = Tr(e^{-\beta(H-i\Omega_E J)}) = Tr(e^{-\beta_+ L_0 - \beta_- \bar{L}_0}). \quad (33)$$

Where, β is the inverse temperature, J is the angular momentum, Ω_E is the Euclidean angular velocity (related to the Minkowskian angular velocity by $\Omega_E = -i\Omega$). In the above equation we have identified the left and right moving temperatures as $\beta_{\pm} = \beta(1 \pm i\Omega_E)$. Notice that the conserved Virasoro charges are the Hamiltonian $H = L_0 + \bar{L}_0$ and the angular momentum $J = L_0 - \bar{L}_0$ with L_0 and \bar{L}_0 being the holomorphic and anti-holomorphic zeroth mode Virasoro generators. Note that this CFT_{1+1} lives on a twisted cylinder and may be obtained by the Euclidean CFT on a complex plane by the following conformal transformation

$$w = \frac{\beta(1 - i\Omega_E)}{2\pi} \log[z], \quad (34)$$

where z denotes the coordinates on the complex plane and w corresponds to the coordinates on the twisted cylinder mentioned above. This leads to the following definition for the entanglement negativity of an extended bipartite quantum system $(A \cup A^c)$ in a CFT_{1+1} at finite temperature and with a conserved angular momentum.

$$\mathcal{E} = \lim_{L \rightarrow \infty} \lim_{n_e \rightarrow 1} \log[\langle \mathcal{T}_{n_e}(-L) \bar{\mathcal{T}}_{n_e}^2(u) \mathcal{T}_{n_e}^2(v) \bar{\mathcal{T}}_{n_e}(L) \rangle_{\beta, \Omega_E}]. \quad (35)$$

In the above expression the subscript β, Ω_E indicates that the four point function has to be evaluated in a CFT_{1+1} on a twisted cylinder. The four point function on a twisted cylinder may be obtained from the four point function on the complex plane, by using the following transformation

$$\langle \mathcal{T}_{n_e}(w_1) \bar{\mathcal{T}}_{n_e}^2(w_2) \mathcal{T}_{n_e}^2(w_3) \bar{\mathcal{T}}_{n_e}(w_4) \rangle_{\beta, \Omega_E} = \prod_{i=1}^4 \left| \frac{dz_i}{dw_i} \right|^{\Delta_{n_e}^i} \langle \mathcal{T}_{n_e}(z_1) \bar{\mathcal{T}}_{n_e}^2(z_2) \mathcal{T}_{n_e}^2(z_3) \bar{\mathcal{T}}_{n_e}(z_4) \rangle_{\mathbb{C}}. \quad (36)$$

$\Delta_{n_e}^i$ in the above equation represents the scaling dimension of the operator at z_i . The four point function in a CFT_{1+1} on the complex plane may be shown to have the following form [9]

$$\langle \mathcal{T}_{n_e}(z_1) \bar{\mathcal{T}}_{n_e}^2(z_2) \mathcal{T}_{n_e}^2(z_3) \bar{\mathcal{T}}_{n_e}(z_4) \rangle_{\mathbb{C}} = \frac{c_{n_e} c_{n_e}^{(2)}}{z_{14}^{2\Delta_{n_e}} z_{23}^{2\Delta_{n_e}^{(2)}}} \frac{\mathcal{F}_{n_e}(x)}{x^{\Delta_{n_e}^{(2)}}}, \quad x \equiv \frac{z_{12} z_{34}}{z_{13} z_{24}}. \quad (37)$$

Notice from above that the four point function is determined only up to a function of the cross-ratio ($x = \frac{z_{12} z_{34}}{z_{13} z_{24}}$). This function denoted here as $\mathcal{F}_{n_e}(x)$ depends on the full operator content of the theory. We compute the required four point function on the twisted cylinder by substituting the four point function on a plane given by eq.(37) in the transformation given by eq.(36). We also identify the points $(w_1, w_2, w_3, w_4) \equiv (-L, u, v, L)$. After obtaining the four point function function, it may then be substituted in eq.(35) to obtain the entanglement negativity as follows

$$\mathcal{E} = \frac{c}{4} \log \left[\frac{\beta_+ \beta_-}{\pi^2 a^2} \sinh \left(\frac{\pi \ell}{\beta_+} \right) \sinh \left(\frac{\pi \ell}{\beta_-} \right) \right] - \frac{\pi c \ell}{2\beta(1 + \Omega_E^2)} + f(e^{-\frac{2\pi \ell}{\beta(1 + \Omega_E^2)}}) + \ln[c_{1/2}^2 c_1], \quad (38)$$

where $\ell = |u - v|$ is length of the subsystem- A and a is UV cut-off for the field theory. The constants c_1 and $c_{\frac{1}{2}}$ may be set to unity by normalizing the two point function of the twist and

the anti-twist fields appropriately. The function $f(x)$ in the above equation is defined in the replica limit as follows

$$f(x) = \lim_{n_e \rightarrow 1} \ln[\mathcal{F}_{n_e}(x)], \quad \lim_{L \rightarrow \infty} x = e^{-\frac{2\pi\ell}{\beta(1+\Omega_E^2)}}. \quad (39)$$

It is to be noted that this function $f(x)$ is undetermined except at the limiting cases $x = 0$ and $x = 1$. The value for this function at these two limits is given by

$$f(1) = 0, \quad f(0) = \lim_{n_e \rightarrow 1} \frac{C_{\mathcal{T}_{n_e} \bar{\mathcal{T}}_{n_e}^2 \bar{\mathcal{T}}_{n_e}}}{c_n^{(2)}}, \quad (40)$$

These expressions for the limiting cases of the function $f(x)$ follow from the argument provided in [9] for the finite temperature scenario. The constants $C_{\mathcal{T}_{n_e} \bar{\mathcal{T}}_{n_e}^2 \bar{\mathcal{T}}_{n_e}}$ and $c_n^{(2)}$ in the equation above are the coefficients of the leading term in the operator product expansion (OPE) of $\mathcal{T}_{n_e}(z_1) \bar{\mathcal{T}}_{n_e}^2(z_2)$ and $\mathcal{T}_{n_e}^2(z_1) \bar{\mathcal{T}}_{n_e}^2(z_2)$ respectively. Following this we make a Wick rotation $\Omega = i\Omega_E$ and re-write the right hand side of eq.(38) in terms of entanglement entropy(S_A) given in eq.(10) and the thermodynamic entropy of subsystem A ($S_A^{th} = \frac{\pi c \ell}{3\beta(1-\Omega^2)}$) which leads us to

$$\mathcal{E} = \frac{3}{2}[S_A - S_A^{th}] + f(e^{-\frac{2\pi\ell}{\beta(1-\Omega^2)}}). \quad (41)$$

We see from the expression above that the entanglement negativity serves as a precise measure of the quantum distillable entanglement at finite temperatures through the removal of the classical/thermal contributions. One can clearly see that when $\Omega = 0$, the above equation reduces to the expected result for the entanglement negativity of a finite temperature CFT_{1+1} obtained in [9]. Note that in the large central charge limit the first two universal terms in the above equation are of $O[c]$ and they dominate over the non-universal term $f(x)$ in eq.(41) which is subleading. The detailed argument for this point involves certain results from the large- c limit of the conformal block expansions and two constraints on the entanglement negativity obtained from quantum information theory (See [27] for details). Hence, in the large- c limit we can ignore the term involving the undetermined function $f(x)$ in the above eq.(41). This leads us to the following expression for the large- c limit of the entanglement negativity of a bipartite system in a CFT_{1+1} with a conserved angular momentum and at a finite non-zero temperature

$$\mathcal{E} = \frac{3}{2}[S_A - S_A^{th}]. \quad (42)$$

This result matches exactly with eq.(28) which was obtained from the bulk computation using our covariant holographic entanglement negativity conjecture. Thus, providing us a verification for our conjecture in the AdS_3/CFT_2 framework.

6 Covariant holographic entanglement negativity of a CFT_{1+1} dual to a rotating extremal BTZ

In this section we proceed to compute the holographic entanglement negativity of a CFT_{1+1} that is dual to extremal rotating BTZ black hole using our covariant prescription and demonstrate that for this case also the entanglement negativity captures the distillable quantum entanglement. The CFTs that are dual to the extremal black holes are very subtle and have many interesting properties. In [36], the authors propose the Kerr-CFT correspondence according to which $(d+1)$ - dimensional extremal Kerr-AdS black holes are dual to a chiral half of a CFT in d dimensions. In the AdS_3/CFT_2 context, it was shown in [37] that the entanglement entropy of a chiral half of a CFT_{1+1} matches exactly with the covariant holographic entanglement entropy

computed from the bulk BTZ black hole using the HRT proposal. This was found to be true irrespective of whether one considers the near horizon metric or the full metric. The metric of the extremal BTZ black hole may be obtained by equating inner and outer radius of horizon i.e $r_+ = r_-$ in eq.(16) as

$$ds^2 = -\frac{(r^2 - r_0^2)^2}{r^2} dt^2 + \frac{r^2 dr^2}{(r^2 - r_0^2)^2} + r^2 (d\phi - \frac{r_0^2}{r^2} dt)^2, \quad (43)$$

where $r_+ = r_- = r_0$ and $J = M = 2r_0^2$ from eq.(17). We first briefly review the method to obtain holographic entanglement entropy of a CFT_{1+1} dual to an extremal BTZ described in [37]. In order to obtain the required geodesic lengths it is possible to make the following coordinate transformation to obtain the metric of the pure AdS_3 space time in the Poincare coordinates

$$w_+ = \phi + t - \frac{r_0}{r^2 - r_0^2}, \quad (44)$$

$$w_- = \frac{1}{2r_0} e^{2r_0(\phi-t)}, \quad (45)$$

$$Z = \frac{1}{\sqrt{r^2 - r_0^2}} e^{r_0(\phi-t)}. \quad (46)$$

Note that these transformations can not be obtained by naively putting $r_+ = r_-$ in eq.(18) and eq.(19). Under the transformation given by eq.(44), the metric of the extremal BTZ black hole in eq.(43) becomes

$$ds^2 = \frac{dw_+ dw_- + dZ^2}{Z^2} \equiv \frac{-dT^2 + dX^2 + dZ^2}{Z^2}. \quad (47)$$

The computation of the length of the spacelike geodesic is similar to the case involving the non extremal black holes discussed earlier, leading to the following expression for the entanglement entropy

$$S_\gamma = \frac{\mathcal{L}_\gamma}{4G_N^{(3)}} = \frac{c}{6} \log \left[\frac{(\Delta X)^2}{\varepsilon_i \varepsilon_j} \right] \quad (48)$$

$$\varepsilon_i = \frac{1}{r_\infty} e^{r_0(\phi_i - t_0)}. \quad (49)$$

If the above expression is re-expressed in the BTZ coordinates then the entanglement entropy of a subsystem- γ in the dual CFT_{1+1} is given by (see [37] for details)

$$S_\gamma = \frac{c}{6} \log \left[\frac{|\phi_i - \phi_j|}{a} \right] + \frac{c}{6} \log \left[\frac{1}{r_0 a} \sinh(r_0 |\phi_i - \phi_j|) \right]. \quad (50)$$

Note that the first term in the above equation resembles the entanglement entropy of a subsystem γ of a zero temperature CFT_{1+1} whereas the second term is the entanglement entropy of the same subsystem in a CFT_{1+1} with an effective temperature $\frac{r_0}{\pi}$. The authors in [37] noted that this has a clear explanation that the left movers of the CFT are in ground state while the right movers have an effective temperature known as the Frolov-Thorne temperature² [38] given by

$$T_{FT} = \frac{1}{\beta_-} = \frac{r_0}{\pi}. \quad (51)$$

² The Frolov-Thorne entropy may be understood as follows. The left and the right moving temperatures of the dual CFT_{1+1} may be expressed in terms of the inner and outer horizon radius of the bulk black string using eq.(17) i.e $\frac{1}{\beta_+} = T_+ = \frac{r_+ - r_-}{2\pi}$ and $\frac{1}{\beta_-} = T_- = \frac{r_+ + r_-}{2\pi}$. When black string becomes extremal the thermal temperature vanishes $T_+ = 0$ but there remains an effective temperature called the Frolov-Thorne temperature $T_{FT} = \frac{r_0}{\pi} = \frac{1}{\beta_-}$. This results in the entropy of the extremal black string in the bulk $s = \frac{r_0}{4G_N^{(3)}}$ which in the dual CFT_{1+1} corresponds to the thermodynamic entropy density $s = \frac{\pi c}{6\beta_-}$.

Therefore, the entanglement entropy in eq.(50) may be re-expressed as

$$S_{[\phi_i, \phi_j]} = \frac{c}{6} \log \left[\frac{|\phi_i - \phi_j|}{a} \right] + \frac{c}{6} \log \left[\frac{\beta_{FT}}{\pi a} \sinh \left(\frac{\pi |\phi_i - \phi_j|}{\beta_{FT}} \right) \right]. \quad (52)$$

Having obtained the required entanglement entropy, we identify the points $(\phi_1, \phi_2, \phi_3, \phi_4) \equiv (-L, u, v, L)$ and the subsystems $A \equiv [u, v]$, $B_1 \equiv [-L, u]$ and $B_2 \equiv [v, L]$ then we substitute all the quantities in the eq.(24) to obtain the covariant holographic entanglement negativity as

$$\mathcal{E} = \frac{c}{4} \log \left[\frac{\ell}{a} \right] + \frac{c}{4} \log \left[\frac{\beta_{FT}}{\pi a} \sinh \left(\frac{\pi \ell}{\beta_{FT}} \right) \right] - \frac{\pi c \ell}{4 \beta_{FT}}. \quad (53)$$

For brevity the above expression may be re-written as

$$\mathcal{E} = \frac{3}{2} [S_A - S_A^{FT}]. \quad (54)$$

Remarkably the above equation indicates towards an extremely interesting result that the covariant holographic entanglement negativity is the difference between the entanglement entropy S_A and the thermodynamic entropy of the subsystem A ($S_A^{FT} = s\ell = \frac{\pi c \ell}{6 \beta_-}$) suggesting that the latter does not contribute to the distillable quantum entanglement and behaves like an effective thermal entropy. Note that this is because the ground state of the extremal black hole is highly degenerate giving rise to an emergent thermodynamic behavior. The Frolov-Thorne entropy is the measure of this degeneracy. In the boundary CFT this corresponds to a thermodynamic entropy with an effective temperature contributing to the entanglement entropy. Therefore it has to be subtracted away like the thermal entropy to obtain the distillable quantum entanglement measured by the entanglement negativity. Thus, here we have demonstrated that once again the covariant holographic entanglement negativity of a CFT_{1+1} dual to an extremal rotating BTZ black hole, captures the distillable quantum entanglement.

7 Entanglement negativity in a CFT_{1+1} with angular momentum and at zero temperature

In this section we compute the entanglement negativity of a zero temperature CFT_{1+1} with a conserved angular momentum and demonstrate that this matches exactly with the bulk result obtained using our conjecture in the previous section. The authors in [37] construct the conformal transformation that maps the points on the complex plane to that on the cylinder where the dual CFT lives by observing how the co-ordinate transformations in eq.(44) and eq.(45) behave as $r \rightarrow \infty$. The authors then use the transformation to show that the entanglement entropy computed from the two point function of the twist and anti twist fields on such a cylinder matches exactly with the holographic entanglement entropy obtained from the bulk extremal BTZ black hole. The behavior of eq.(44) and eq.(45) as $r \rightarrow \infty$ suggests that the above mentioned conformal transformation is given by

$$w = z, \quad (55)$$

$$\bar{w} = \frac{\beta_-}{2\pi} \log \left[\frac{2\pi \bar{z}}{\beta_-} \right]. \quad (56)$$

As discussed in section-4, from eq.(36) and eq.(37), the four point function in the CFT_{1+1} on the cylinder is related that on the complex plane by

$$\begin{aligned} \langle \mathcal{T}_{n_e}(w_1) \overline{\mathcal{T}}_{n_e}^2(w_2) \mathcal{T}_{n_e}^2(w_3) \overline{\mathcal{T}}_{n_e}(w_4) \rangle_{\beta_-} &= \prod_{i=1}^4 \left| \frac{dz_i}{dw_i} \right|^{\Delta_{n_e}^i} \langle \mathcal{T}_{n_e}(z_1) \overline{\mathcal{T}}_{n_e}^2(z_2) \mathcal{T}_{n_e}^2(z_3) \overline{\mathcal{T}}_{n_e}(z_4) \rangle_{\mathbb{C}}, \\ &= \prod_{i=1}^4 \left(\frac{dz_i}{dw_i} \frac{d\bar{z}_i}{d\bar{w}_i} \right)^{\frac{\Delta_{n_e}^i}{2}} \frac{c_{n_e} c_{n_e}^{(2)}}{z_{14}^{2\Delta_{n_e}} z_{23}^{2\Delta_{n_e}^{(2)}}} \frac{\mathcal{F}_{n_e}(x)}{x^{\Delta_{n_e}^{(2)}}}. \end{aligned} \quad (57)$$

We identify the points at which the four point function has to be evaluated as $(w_1, w_2, w_3, w_4) \equiv (-L, u, v, L)$, therefore from eq.(56) we have $(z_1, z_2, z_3, z_4) \equiv (-L, u, v, L)$ and $(\bar{z}_1, \bar{z}_2, \bar{z}_3, \bar{z}_4) \equiv \frac{\beta_-}{2\pi}(e^{-\frac{2\pi L}{\beta_-}}, e^{\frac{2\pi u}{\beta_-}}, e^{\frac{2\pi v}{\beta_-}}, e^{\frac{2\pi L}{\beta_-}})$. After this identification, we compute the four point function in eq.(57) and substitute it in eq.(30) to obtain the entanglement negativity as

$$\mathcal{E} = \frac{c}{4} \log \left[\frac{\ell}{a} \right] + \frac{c}{4} \log \left[\frac{\beta_-}{\pi a} \sinh \left(\frac{\pi \ell}{\beta_-} \right) \right] - \frac{\pi c \ell}{4 \beta_-} + f(e^{-\frac{\pi \ell}{\beta_-}}). \quad (58)$$

As discussed in section-5, in the large- c limit the entanglement negativity receives the leading contribution from the universal terms which of $O[c]$ (first three terms in the above equation) and the non universal term which is the function $f(x)$ is subleading. Furthermore, identifying $\beta_- = \beta_{FT}$ we obtain the large- c limit of the entanglement negativity of a zero temperature CFT_{1+1} with a conserved angular momentum as

$$\mathcal{E} = \frac{3}{2} [S_A - S_A^{FT}]. \quad (59)$$

The above expression for the entanglement negativity exactly matches with eq.(53) obtained from the dual bulk extremal rotating BTZ black hole using our covariant holographic entanglement negativity conjecture. Thus, we have verified our conjecture by demonstrating that it exactly reproduces the large c limit of the entanglement negativity of CFT_{1+1} in the context of AdS_3/CFT_2 involving the bulk rotating BTZ black holes.

8 Summary and Conclusion

In this paper, we have proposed a covariant conjecture for the holographic negativity of a d dimensional CFT dual to a non-static AdS_{d+1} space time in the AdS_{d+1}/CFT_d scenario. We made use of our proposal to evaluate the entanglement negativity of a $(1+1)$ dimensional CFT which is dual to a $(2+1)$ dimensional rotating BTZ black hole. We computed this quantity in the AdS_3/CFT_2 context for a finite temperature CFT_{1+1} when the bulk is a non extremal rotating BTZ black hole and then for the case of zero temperature CFT_{1+1} involving a bulk extremal rotating BTZ black hole. Remarkably, we see that at finite temperatures when the bulk is a non-extremal black hole the entanglement negativity removes the thermal contribution measuring only the distillable quantum entanglement. On the other hand for the case involving a CFT_{1+1} dual to an extremal black hole, we demonstrated that the entanglement negativity subtracts away the Frolov-Thorne entropy of the subsystem suggesting that the extremal black holes are dual to a chiral half of a CFT.

Our conjecture provides a new tool to study the time-dependent dynamical processes involving entanglement evolution in quantum field theories with conformal symmetry in arbitrary dimensions. Therefore it will find applications in diverse areas such as quantum quenches, thermalization, high T_c superconductors in condensed matter systems and the long standing problem of the black hole information paradox and the related firewall problem.

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